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Effect of heat transfer on the performance of thermoelectric generators

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Abstract

The power output and efficiency expressions for thermoelectric (semiconductor) generators which is composed of multi-elements are derived with considerations of heat transfer irreversibility in the heat exchangers between the generator and the heat reservoirs. Numerical examples are provided. The effects of heat transfer and the number of elements on the performance are analyzed.

Keywords: Thermoelectric generator; Multi-elements device; Heat transfer effect; Finite time thermodynamics

1. Introduction

Several authors have applied the finite time thermodynamics or entropy generation minimization [1–3] to the analysis of thermoelectric generator [4–10]. They analyzed the effect of finite-rate heat transfer between the thermoelectric device and its external heat reservoirs on the performance of the single-element thermoelectric generator. In practice, a thermoelectric generator is composed of many fundamental thermoelectric elements. It is a multi-element device. In the present paper we will investigate the characteristics of a multi-element thermoelectric generator with the irreversibility of finite-rate heat transfer, Joulean heat inside the thermoelectric device, and the heat leak through the thermoelectric couple leg. The expressions for power output and efficiency of the multi-element thermoelectric generator are derived algebraically and illustrated numerically.

2. Power and efficiency analyses

Real thermoelectric generator is composed of many thermoelectric elements, which is shown in Fig. 1. Each element is composed of P-type and N-type semiconductor legs. The thermoelectric element is assumed to be insulated, both electrically and thermally, from its surroundings, except at the junction-reservoir contacts. The internal irreversibility is caused by Joulean electrical resistive loss and heat conduction loss through the semiconductors between hot and cold junctions. The Joulean loss generates an internal heat I^2R, where R is the total internal electrical resistance of the semiconductor couple and I is the electrical current generating from the couple. The conduction heat loss is $K(T_{WH} - T_{WL})$, where K is the thermal conductance of the semiconductor couple, T_{WH} is the hot junction temperature, and T_{WL} is the cold junction temperature. The external irreversibility is caused by finite rate heat transfer, i.e., the temperature differences $(T_H - T_{WH})$ and $(T_{WL} - T_L)$. Also $(K_1F_1)_i$ and $(K_2F_2)_i$ in Fig. 1 are the heat conductances in the hot- and cold-side heat exchangers for ith thermoelectric element, where K_1 and F_1 are the heat transfer coefficient and the heat transfer surface area in the hot-side heat exchanger for ith thermoelectric element, and K_2 and F_2 are the heat transfer coefficient and the heat transfer surface area in the cold-side heat exchanger for ith thermoelectric element. Therefore, for each element, we have the rate (Q'_H) of heat transfer from the heat source T_H to the element hot junction at temperature T_{WH}, and the rate (Q'_L) of heat transfer from the element cold junction at temperature T_{WL} to the heat sink T_L as, respectively,

$$Q'_H = \left[\alpha T_{WH} I - 0.5 I^2 R + K(T_{WH} - T_{WL})\right] = (K_1F_1)_i (T_H - T_{WH}) \tag{1}$$

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Nomenclature

F_1	heat transfer surface area in the hot-side heat exchanger	m
F_2	heat transfer surface area in the cold-side heat exchanger	m
I	electrical current generating from the semiconductor couple	A
I_p	optimal electrical current generating from the semiconductor couple at maximum power point	A
$I\eta$	optimal electrical current generating from the semiconductor couple at maximum efficiency point	A
K	thermal conductance of the semiconductor couple	$W\cdot m^{-1}\cdot K^{-1}$
K_1	heat transfer coefficient in the hot-side heat exchanger	$W\cdot m^{-1}\cdot K^{-1}$
K_2	heat transfer coefficient in the cold-side heat exchanger	$W\cdot m^{-1}\cdot K^{-1}$
n	number of the thermoelectric	
P	elements power output	W
P_{max}	maximum power output	W
P_η	power output at maximum efficiency point	W
Q_H	heat transfer from the heat source to the hot junction at temperature	W
Q_L	heat transfer from cold junction to heat sink	W
R	total internal electrical resistance of the semiconductor couple	Ω
R_L	electrical resistance of the external load	Ω
T_H	temperature of the heat source	K
T_L	temperature of the heat sink	K
T_{WH}	temperature of the hot junction	K
T_{WL}	temperature of the cold junction	K

Greek symbols

α_N	Seebeck coefficient of the N-type semiconductor leg	$V\cdot K^{-1}$
α_P	Seebeck coefficient of the P-type semiconductor leg	$V\cdot K^{-1}$
η	thermal efficiency	
η_{max}	maximum thermal efficiency	
η_P	thermal efficiency at maximum power point	

$$Q'_L = \left[\alpha T_{WL} I + 0.5 I^2 R + K(T_{WH} - T_{WL})\right] = (K_2F_2)_i (T_{WL} - T_L) \quad (2)$$

where $\alpha = \alpha_P - \alpha_N$□$\alpha_P$ and α_N are the Seebeck coefficients of the P- and N-type semiconductor legs.

For a real thermoelectric generator composed of n identical thermoelectric elements, we have its rate of absorbing heat (Q_H) and rate of releasing heat (Q_L) as, respectively:

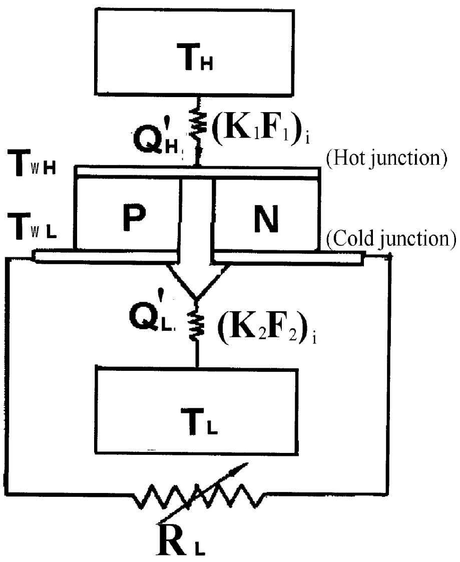

Fig. 1. Thermoelectric generator element.

$$Q_H = K_1F_1(T_{HC} - T_H) = nQ'_H \quad (3)$$

$$Q_L = K_2F_2(T_L - T_{LC}) = nQ'_L \quad (4)$$

where K_1F_1 and K_2F_2 are the heat conductances (product of heat transfer coefficient and heat transfer surface area) of heat exchangers between the hot and cold junctions of the thermoelectric generator and their respective reservoirs.

Combining Eqs. (1)–(4) yields:

$$T_{WH} = \Big(\big(K_1F_1T_H/n + 0.5I^2R\big)\big)(K_2F_2/n + \alpha I) + K\big(K_1F_1T_H/n + K_2F_2T_L/n + I^2R\big) \times \Big((K_1F_1/n + \alpha I)(K_2F_2/n - \alpha I) + K(K_1F_1 + K_2F_2)/n\Big)^{-1} \quad (5)$$

$$T_{WL} = \Big(\big(K_2F_2T_L/n + 0.5I^2R\big)\big)(K_1F_1/n + \alpha I) + K\big(K_1F_1T_H/n + K_2F_2T_L/n + I^2R\big) \times \Big((K_1F_1/n + \alpha I)(K_2F_2/n - \alpha I) + K(K_1F_1 + K_2F_2)/n\Big)^{-1} \quad (6)$$

$$Q_H = K_1F_1\big\{I^3R\alpha/2 - I^2\big[T_H\alpha^2 + K_2F_2R/(2n) + KR\big] + IK_2F_2T_H\alpha/n + KK_2F_2(T_H - T_L)/n\big\} \times \Big((K_1F_1/n + \alpha I)(K_2F_2/n - I\alpha) + K(K_1F_1 + K_2F_2)/n\Big)^{-1} \quad (7)$$

$$Q_L = K_2F_2\{I^3R\alpha/2 + I^2[T_L\alpha^2 + K_1F_1R/(2n) + KR] + IK_1F_1T_L\alpha/n + KK_1F_1(T_H - T_L)/n\} \times ((K_1F_1/n + \alpha I)(K_2F_2/n - \alpha I) + K(K_1F_1 + K_2F_2)/n)^{-1} \quad (8)$$

The total power output (P) and the efficiency (η) of the multi-element thermoelectric generator are as follows:

$$P = Q_H - Q_L \quad (9)$$

$$\eta = 1 - Q_L/Q_H \quad (10)$$

Combining Eqs. (7)–(10) gives:

$$P = (I^3(K_1F_1 - K_2F_2)R\alpha/2 - I^2[(K_1F_1T_H + K_2F_2T_L)\alpha^2 + K_1F_1K_2F_2R/n + (K_1F_1 + K_2F_2)KR] + IK_1F_1K_2F_2\alpha(T_H - T_L)/n) \times ((K_1F_1/n + \alpha I)(K_2F_2/n - \alpha I) + K(K_1F_1 + K_2F_2)/n)^{-1} \quad (11)$$

$$\eta = (I^3(K_1F_1 - K_2F_2)R\alpha/2 - I^2[(K_1F_1T_H + K_2F_2T_L)\alpha^2 + K_1F_1K_2F_2R/n + (K_1F_1 + K_2F_2)KR] + IK_1F_1K_2F_2\alpha(T_H - T_L)/n) \times (K_1F_1\{I^3R\alpha/2 - I^2[T_H\alpha^2 + K_2F_2R/(2n) + KR] + IK_2F_2T_H\alpha/n + KK_2F_2(T_H - T_L)/n\})^{-1} \quad (12)$$

Eqs. (11) and (12) are the major results of this paper. They reflect the effects of heat transfers (K_1F_1 and K_2F_2), heat reservoir temperatures (T_H and T_L), internal heat conductance (K), internal electrical resistance (R), Seebeck coefficient (α), working electrical current (I) and number of thermoelectric elements (n) on the power output (P) and efficiency (η) of a multi-element thermoelectric generator.

If $n = 1$, Eqs. (11) and (12) become the finite time analysis results of single-element thermoelectric generator [4,5,9,10].

If $K_1F_1 = K_2F_2 \to \infty$, $T_{WH} = T_H$ and $T_{WL} = T_L$, Eqs. (11) and (12) become theresults of conventional non-equilibrium thermodynamic analysis

$$P = [\alpha(T_H - T_L)I - I^2R]n \quad (13)$$

$$\eta = \frac{\alpha(T_H - T_L)I - I^2R}{\alpha T_H I - 0.5I^2R + K(T_H - T_L)} \quad (14)$$

In this case, P is dependent on n but η is independent of n. If $n = 1$, Eqs. (13) and (14) become the results of references [11,12]. However, $T_{WH} = T_H$ and $T_{WL} = T_L$ need infinite heat transfer surface area, its specific power output $P/(F_1 + F_2)$ is zero [7].

Eqs. (11) and (12) show that there exist two important points: a maximum power point (P_{max}) with the corresponding efficiency η_P and the optimal electrical current I_P, and a maximum efficiency point (η_{max}) with the corresponding power output P_η and the optimal electrical current I_η. From the point of view of finite time thermodynamic optimization (the compromise optimization between power output and the efficiency), the design parameters optimal region of the thermoelectric generator should be:

$$P_\eta \leqslant P \leqslant P_{max} \quad (15)$$

$$\eta_P \leqslant \eta \leqslant \eta_{max} \quad (16)$$

For the case of infinite rate heat transfer, I_p, P_{max}, η_P, I_η, η_{max} and P_η could be obtained analytically [11]. However, for the general case of finite rate heat transfer, they should be obtained numerically.

3. Numerical examples

Taking $T_H = 600$ K, $T_L = 300$ K, $\alpha = 2.3 \times 10^{-4}$ V $\cdot$K^{-1}, $R = 1.4 \times 10^{-3}$ Ω, $K = 1.5 \times 10^{-2}$ W$\cdot$K$^{-1}\cdot$m^{-1}, $K_1F_1 = 4$ W$\cdot$K^{-1} and $K_2F_2 = 1$ W$\cdot$K^{-1}, the power versus current and efficiency versus current characteristics are calculated as shown in Figs. 2 and 3. These parameters are of a product, IEC1-039018 thermoelectric generator, of Shanghai Jin Wei Thermo-electricity Co. Ltd. [15].The dashed lines represent the performance of infinite rate heat transfer thermoelectric generator. Figs. 2 and 3 show that the heat transfer irreversibility does affect the power output and the efficiency of the thermoelectric generator. This effect must be considered in the performance analysis. The power output with considering the heat transfer effect is about half of the power output without considering the heat transfer effect in the case of $n = 10$. For the fixed heat conductance (K_1F_1 and K_2F_2), the maximum electrical current corresponding to zero power output and zero efficiency, the optimal current corresponding to the maximum power output, the optimal current corresponding to the maximum efficiency, and the efficiency decrease with the increase of number (n) of thermoelectric elements. However, there exists power extremal with respect to number (n) of thermoelectric elements, that is, there exists an optimal n corresponding to maximum power output. Therefore, there exists a double maximum power with the optimal current and the optimal number of thermoelectric elements.

In the conventional analysis without considering heat transfer effect, the power output is a monotonic (linear) increasing function of n, and the efficiency is independent of n. Hence, the effect of heat transfer irreversibility increases when n increases for the fixed heat conductance.

In order to analyze the effect of heat conductance on the performance of thermoelectric generator, we calculate

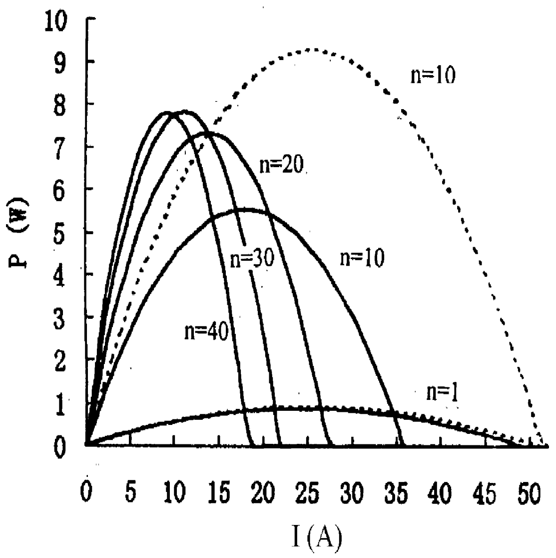

Fig. 2. Power vs. current and number of elements ($K_1F_1 = 4$ W·K^{-1}, $K_2F_2 = 1$ W·K^{-1}).

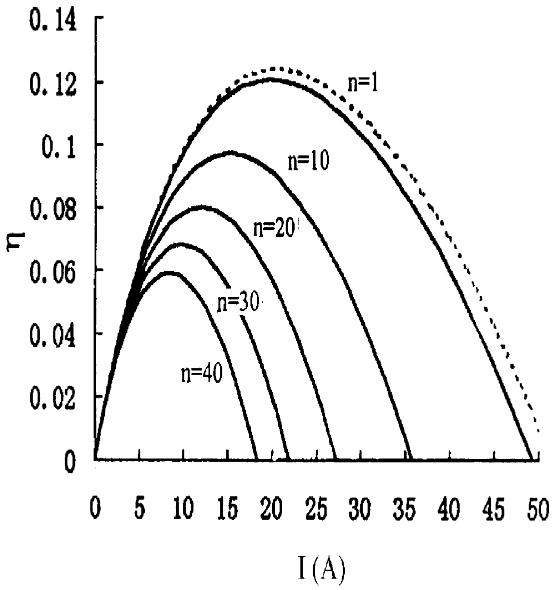

Fig. 3. Efficiency vs. current and number of elements ($K_1F_1 = 4$ W·K^{-1}, $K_2F_2 = 1$ W·K^{-1}).

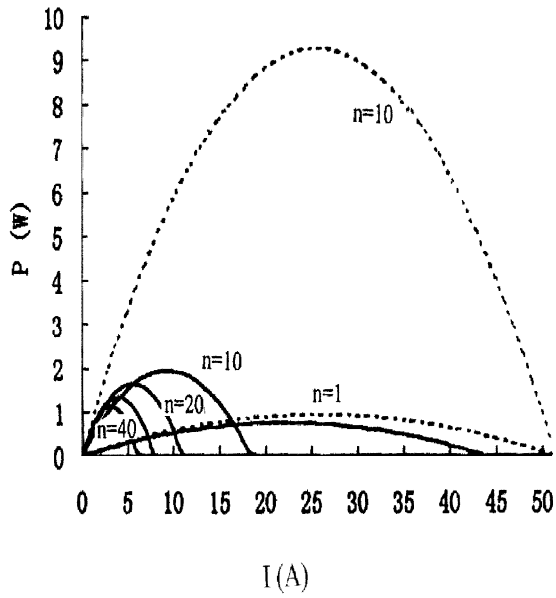

Fig. 4. Power vs. current and number of elements ($K_1F_1 = 1$ W·K^{-1}, $K_2F_2 = 0.25$ W·K^{-1}).

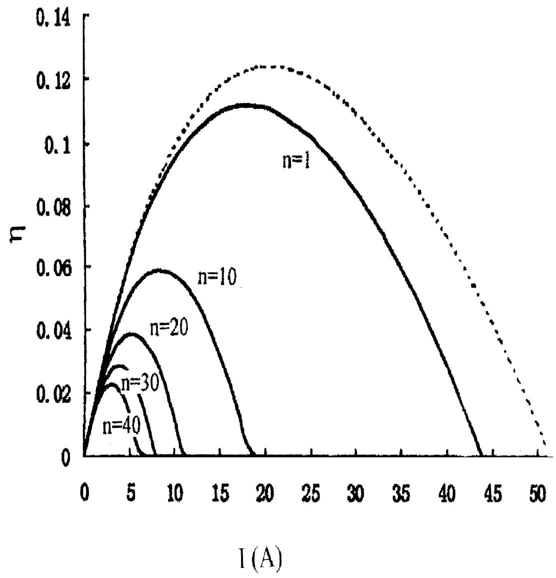

Fig. 5. Efficiency vs. current and number of elements ($K_1F_1 = 1$ W·K^{-1}, $K_2F_2 = 0.25$ W·K^{-1}).

the performance by taking $K_1F_1 = 1$ W·K^{-1}, $K_2F_2 = 0.25$ W·K^{-1} and as shown in Figs. 4 and 5 (the other parameters are the same as those for Figs. 2 and 3). Comparing Figs. 2 and 4, and Figs. 3 and 5, we know that the heat conductance affects the power and efficiency obviously. With the decrease of heat exchanger performance, the maximum current, the optimal current corresponding to the maximum power output, the optimal current corresponding to the maximum efficiency and the optimal number of thermoelectric elements corresponding to the maximum power output decrease.

The numerical results show that the power vs. efficiency characteristics of real multi-element thermoelectric generator is a loop-shaped curve. This is consistent with those of a generalized irreversible Carnot engine theoretical model established by Chen et al. [13,14].

4. Conclusion

The results of this paper show that the heat transfer irreversibility does affect the performance of thermoelectric generator. This effect must be considered in the analysis. For the real generator composed of multi-element, the number of thermoelectric elements affects the performance too. The optimal current and the optimal number of thermoelectric elements must be choose in the points of view of the compromise optimization between power output and efficiency in order to obtain the best performance.

The performance optimization of thermoelectric generator could be carried out further. The optimization includes two aspects, i.e., the internal optimization and the external optimization. The former is to optimize the thermoelectric couple leg size of each element to minimize (KR). The latter is to optimize the distribution of heat transfer surface areas or heat conductance for the two heat exchangers and to optimize the external load (R_L) and internal resistance (R) matching. Those will be the subjects of a future paper.

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